Abstract: Experiments have been performed to assess the efficiency of mechanical mixers in destratifying water bodies. The effects of Reynolds number, physical scale, and Richardson number were all considered. The results demonstrated that the variation of mixing efficiency with Richardson number was well described by two different power-law regimes. At low Richardson numbers, the efficiency increased with increasing Richardson number. Above a critical value, however, the efficiency decreased rapidly with further increases in Richardson number. The experiments further showed that, over the range of Reynolds numbers considered, the results collapsed fairly well. Finally, experiments at different physical scales showed reasonable agreement. The main conclusion of the present technical note is that the destratification efficiency of mechanical mixers can be well parameterized by an overall Richardson number. The results, however, are specific to the particular geometric configuration studied. It is hoped that the present technical note will help to guide future studies of destratification systems in small lakes and reservoirs and fluid mixing in the chemical and process industries.

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CE Database subject headings: Stratified flow; Reservoirs; Stratification; Water quality; Turbulence; Mixing.

Introduction

Mixing processes are of interest in hydraulic engineering as they redistribute tracers including heat, contaminants and nutrients, and fine sediments. Given the large length and velocity scales typical of hydraulic flows, turbulent diffusion is the primary mechanism for this redistribution. The presence of vertical density gradients, or stratification, has a strong impact on mixing in turbulent flows. Excellent reviews on the topic are provided by Linden (1979) and Fernando (1991). Conceptually, stratification serves to reduce, or completely suppress, turbulent mixing as a portion of the turbulent kinetic energy must be expended in overcoming gravitational effects.

Stratification in water bodies is a matter of significant practical importance. Stephens and Imberger (1993) discuss how stratification, often driven by strong solar heating of near-surface waters in the summer, limits the transport of oxygen to hypolimnetic waters. This, in turn, will lead to anoxic conditions in these lower waters. There are two popular methods for destratifying water bodies—bubble plumes and mechanical mixers. Antenucci et al. (2003) point out that bubble plumes have the advantages of (1) being able to cover a greater horizontal extent and (2) having shore-based mechanical parts. Mechanical mixers, on the other hand, have the advantage of being able to directly access and entrain oxygen-rich surface waters.

Regarding mechanical mixers, there have been numerous laboratory and field studies in the past that have assessed their performance and have considered how to predict prototype performance (Busnaina et al. 1981; Robinson et al. 1982; Vandermeulen 1992; Stephens and Imberger 1993; Kirke and Gezawy 1997; Milstein et al. 2001). The work of Stephens and Imberger (1993) is the most relevant to the present technical note as it represents the first systematic and thorough laboratory investigation of the destratification (by mechanical mixers) process. Their experiments considered both two-layer and linear density profiles and their results were parameterized by an “impeller number.” Maximum mixing efficiencies were found to compare favorably with bubble plume destratification systems.

The goal of the present technical note is to more thoroughly investigate the mixing efficiency of impeller-driven turbulence in stratified flows. The objectives are to understand the effects of the overall Richardson number, the Reynolds number, and the physical scale of the experiment. To help isolate these relationships, the present study is restricted to a single geometrical configuration. It is recognized that the obtained results are specific to the particular configuration being studied. Of greatest interest, therefore, are the general trends and the identification of the relative importance of the above-mentioned parameters.

Experimental Facilities and Procedures

The present experiments were carried out in a series of un baffled cylindrical tanks, a schematic of which is shown in Fig. 1(a). The inner diameters \(D_f\) of the tanks were 29.1, 44.3, and 59.7 cm. Two-layer stratification in the tanks was created by first filling the tank with fresh water to a depth of \(D_f/2\). Then, using a bottom-mounted diffuser plate, salt water was introduced until the total fluid depth was equal to \(D_f\). Typically, the interface between the...
two layers had a thickness on the order of 1 cm. The salt water was dyed with green food coloring in order to provide sharp visual contrast between the two layers. Salt water densities were measured with high-precision hydrometers (specific gravity accurate to ±0.0005). For the range of experiments conducted, lower layer densities ranged from 1,005 to 1,200 kg m⁻³.

Mixing in the tank was provided by the rotation of a standard Rushton turbine. Rushton impellers, which are radial-flow devices, were chosen due to the availability of a set of custom-made, geometrically scaled impellers. Following standard Rushton turbine/mixing tank conventions, the impeller diameter \(D\) for each experiment was one third the tank diameter and the total fluid depth was equal to \(D_p\) as described earlier. In all experiments, the impeller was located at the vertical midpoint of the water column (the initial location of the pycnocline) and rotation was controlled by programmable mixers (Lightnin, Inc.).

An individual experiment began with a sharp, two-layer density profile as described earlier. Upon commencement of impeller rotation, the tank was subsequently characterized by a three-layer density profile [Fig. 1(b)], with the middle layer of mixed fluid steadily growing at the expense of the remaining fresh and salt water layers. As suggested by Fig. 1(b), the mixed layer was observed to grow symmetrically about the initial interface. Limited profiles with a conductivity/temperature probe verified this observation. Eventually, the tank would become fully destratified, as shown in Fig. 1(c).

To measure the temporal evolution of the tank density profile, the tank was backlit with incandescent light and a ruled scale was placed on the front of the tank. A video recorder was then used to record the experiment. Subsequently, the video was reviewed and the locations of the interfaces among the three layers were recorded as a function of time. Due to slight fluctuations of the interface, it was estimated that the maximum experimental uncertainty in locating the interface was ±3 mm.

The experimental matrix was designed to include three tank sizes, four Reynolds numbers, and eight Richardson numbers, for a total of 96 trials. Ultimately, the matrix was constrained by several factors, including mixer speed and the saturation point of salt. In summary, therefore, a total of 73 experimental trials were carried out, as summarized in Table 1.

For mechanical mixers, the Reynolds number can be defined as

\[
R = \frac{N D^3}{\nu} \tag{1}
\]

where \(\nu\) = fluid kinematic viscosity and \(N\) = rotational rate in revolutions per second. The general form of an overall Richardson number is

![Fig. 1. Schematic representation of experimental setup: (a) prior to commencement of experiment; (b) during experiment; and (c) at the conclusion of experiment](image-url)
Table 1. (Continued.)

<table>
<thead>
<tr>
<th>N (rpm)</th>
<th>R</th>
<th>( g' ) (m/s²)</th>
<th>Ri₀</th>
<th>P (W)</th>
<th>( \eta )</th>
<th>( T_m ) (s)</th>
</tr>
</thead>
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<tr>
<td>Large tank ((D_T=59.7\text{ cm}, ; D=19.9\text{ cm}))</td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td>2.060</td>
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<td>0.164</td>
<td>0.01</td>
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<td>0.162</td>
<td>0.04</td>
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</tr>
<tr>
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<td>0.45</td>
<td>0.05</td>
<td>0.356</td>
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<tr>
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</tr>
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<td>0.03</td>
<td>0.352</td>
<td>0.24</td>
<td>36</td>
</tr>
</tbody>
</table>

\[ \text{Ri}_0 = \frac{g' L}{U^2} \]  
\[ g' = \frac{g \Delta \rho}{\rho} \]  
\[ \text{Ri}_o = \frac{g' H}{R^2 \omega^2} \]

Experimental Results

Fig. 2 shows the temporal evolution of the location of the upper density interface. Given the observed symmetry, as discussed previously, it is sufficient to consider results for this upper interface only. The results in Fig. 2 are for the case of \( D_T=29.1 \text{ cm} \) and \( R=8,090 \) and the error bars reflect the uncertainty in determining the location of the interface. Time has been scaled by the rotational period, i.e., \( t^*=tN \), and the vertical location of the upper interface has been scaled by the tank half-height, i.e., \( z^*_i = z_i/(D_T/2) \).

At low values of \( \text{Ri}_o \), it is found that the time history of \( z^*_i \) is very well described by a power-law relationship, given by

\[ z^*_i = a (t^*)^b \]

with typical \( R^2 \) values in excess of 0.98. As \( \text{Ri}_o \) and correspondingly the stabilizing buoyancy force, increases, it is, not surprisingly, found that the rate of mixing steadily decreases. This is reflected in Table 2, where \( b \) as \( \text{Ri}_o \)

Finally, beyond some threshold value of \( \text{Ri}_o \), there is a distinct change in the mixing behavior in the tank. At early times for these runs, the behavior is much as it was for the lower \( \text{Ri}_o \) trials. At later times, however, the slope of the power-law behavior changes markedly and very long times are now required to fully destratify the tank. The results from the other tank size/Reynolds number combinations are similar to those shown in Fig. 2.

Table 2. Power-Law Coefficients Describing [Eq. (4)] the Evolution of the Stratification in the Mixing Tank; for the Higher \( \text{Ri}_o \) Cases, the Given Coefficients Describe the Initial Stage of Evolution Only

<table>
<thead>
<tr>
<th>( \text{Ri}_o )</th>
<th>( a )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.026</td>
<td>0.086</td>
<td>0.72</td>
</tr>
<tr>
<td>0.040</td>
<td>0.094</td>
<td>0.67</td>
</tr>
<tr>
<td>0.057</td>
<td>0.092</td>
<td>0.63</td>
</tr>
<tr>
<td>0.075</td>
<td>0.073</td>
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</tr>
<tr>
<td>0.10</td>
<td>0.085</td>
<td>0.57</td>
</tr>
<tr>
<td>0.15</td>
<td>0.089</td>
<td>0.50</td>
</tr>
<tr>
<td>0.27</td>
<td>0.093</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Fig. 2. Variation of scaled interface location with scaled time and overall Richardson number for the \( R=8,090, \; D_T=29.1 \text{ cm} \) trial. The solid lines represent power-law fits for the different experiments.
Mixing Efficiency

As discussed in the Introduction, a mixing efficiency, denoted by \( \eta \), quantifies the fraction of mechanical energy input that is consumed by the increase in potential energy of a stratified system, i.e.

\[
\eta = \frac{\text{rate of potential energy increase}}{\text{rate of energy input}}
\]  

(6)

For the present experiments, recalling the power-law behavior discussed in the previous section, it is clear that the rate of potential energy increase is not constant in time. Rather, the rate of mixing is initially rapid and slows monotonically over the course of an individual trial. Although there are many possible measures of mixing (initial mixing rate, etc.) in the tank, the “average” rate of potential energy increase was adopted as a consistent measure for the present study. It is straightforward to show that, for the equal-depth, two-layer system considered here, this average rate is given by

\[
\eta = \frac{\Delta \rho \frac{g \pi D^3}{32}}{T_m}
\]  

(7)

In this expression, \( T_m \) = mixing time, or the time that it takes to fully mix the tank. For the present experiments, \( T_m \) (see Table 1) ranged from less than 1/2 min to over 1 h.

The rate of energy input, or power \( P \), was calculated from

Fig. 3. Mixing efficiency for the (a) small tank; (b) medium tank; (c) large tank; and (d) combined results. The solid lines represent power-law fits to the low \( Ri_0 \) data and the dotted lines represent power-law fits to the high \( Ri_0 \) data.
\[ N_p = \frac{P}{\rho N^3 D^5} \]  

where \( N_p \) = power number of the impeller. The power number of a Rushton turbine, for both baffled and un baffled tanks, is well established in the literature. For un baffled tanks at high Reynolds number, \( N_p \rightarrow 1 \). For the present study, \( N_p \) values were interpolated from the data of Rushton et al. (1950, Fig. 15) and ranged from 1.24 to 1.40. Other studies have measured the power consumption directly by measuring the voltage drop over the mixer (Stephens and Imberger 1993) or by measuring the torque on the mixer shaft (Karcz and Major 1998).

The mixing efficiencies for the three different tanks are shown in Figs. 3(a–c). First of all, the similarity observed in the results from the different tanks suggests the lack of any substantial systematic influence of physical scale. Examining the results from any one tank, it is also observed that, although there is some scatter, there is no strong systematic variation with the Reynolds number. This Reynolds number invariance agrees well with well-established consensus (Rushton et al. 1950; Rushton and Oldshue 1953) that mixer flows are fully turbulent for \( R > 10^4 \). Next, it is noted that the relationship between \( \eta \) and \( R_l \) follows two distinct power-law regimes. At low \( R_l \), the efficiency initially increases with increasing \( R_l \), beyond some threshold value of \( R_l \), observed to be 0.1, the efficiency decreases rapidly with further increases. In Fig. 3(d), the results from all of the tank sizes are aggregated and power-law coefficients are determined for the complete ensemble of experiments. These coefficients are 0.37 and \(-3.1\) \((R^2\) values of 0.53 and 0.80 for the fit equations) for the low and high \( R_l \) regimes, respectively.

Regarding uncertainty in the mixing efficiency, the calculations include uncertainties in the layer density and the vertical location of the interface. Additionally, they include uncertainty in \( N_p \). Although Rushton et al. (1950) do not provide values, visual inspection of their results suggests a value of about 10%. It is estimated that these combined experimental uncertainties result in an overall uncertainty of 10–20% for \( \eta \).

**Discussion**

The observed relationship between efficiency and Richardson number, with a distinct maximum in efficiency, is intuitive. For, as \( R_l \rightarrow 0 \), there will be nothing to mix in the tank and the efficiency must therefore go to zero. On the other hand, as \( R_l \rightarrow \infty \), the stratification becomes so strong as to completely suppress the turbulence, again yielding an efficiency of zero. Verification of the hypothesized relationship between these variables has attracted wide attention in the laboratory. Many different turbulence generating mechanisms have been considered, including towed grids (Rehmann and Koseff 2004), breaking internal waves (Ivey and Nokes 1989), and others. Linden (1979) provides a thorough review of the topic and demonstrates a definite peak (Fig. 4 of that paper) in the efficiency curve.

**Comparison with Previous Results**

It is of interest to briefly compare the present results with those of Stephens and Imberger (1993). With the exception of their study, most previous studies of destratification by mechanical mixers did not attempt to explore the relationship among flow characteristics, stratification characteristics, and mixing efficiency. Stephens and Imberger (1993) considered axial flow impellers of several sizes, and both linear and two-layer stratifications were considered. For their two-layer experiments, the mixing efficiency was plotted against an impeller number, of their own definition. Aside from the incorporation of information about the respective layer depths, their impeller number is approximately the reciprocal of the overall Richardson number, as given in Eq. (4). From their tabulated data (Table 1 of that paper), the most complete data set is one with \( D = 15 \) cm. Those data, which include runs at many different rotational speeds, were extracted and recast in terms of \( R_l \) by the present writers, and the resultant efficiency curve is shown in Fig. 4.

These data capture the main feature of the present experiments, namely a peak in the efficiency curve, with reasonable adherence to power-law behavior on each side of the peak. There are many differences in the details of the two data sets, which is not surprising given the many differences in the configurations. In particular, the calculated efficiencies and the threshold, or critical, value of \( R_l \) are markedly different from the present study.

**Concluding Remarks**

The major contribution of this technical note is a tightly controlled set of experiments on the mixing efficiency of stirred stratified flows. Experimental trials were carried out in three geometrically similar tanks of differing size. It was shown that the effect of tank size was relatively small. This provides some confidence in scaling up laboratory results to a prototype-scale application. Second, it was shown that, for Reynolds numbers greater than \( O(10^5) \), the results were \( R \) invariant. This is a useful result as prototype-scale flows will undoubtedly be of equal or higher \( R \) than the present experiments. For example, the field test described by Vandermeulen (1992) had \( R \approx 2.5 \times 10^6 \). Having the present experimental results in the fully turbulent regime therefore helps to support the extrapolation to higher values.

Most significantly, however, the present work has clearly
demonstrated that the mixing efficiency and the overall Richardson number are related by a power-law relationship. Moreover, this relationship is broken down into two regimes. At low values of $\text{RI}_o$, the efficiency increases with increasing $\text{RI}_o$. Beyond $\text{RI}_o \sim 0.1$, however, the efficiency rapidly decreases with increasing $\text{RI}_o$. This result is supported by physical arguments as has been demonstrated previously in other stratified turbulence experiments.

The limitation of the present work is that it considers only a single geometrical configuration (equal layer depths, fixed impeller-to-tank diameter ratio, etc.). This narrow scope is justified on two grounds. First, the specific configuration considered was chosen in order to leverage available resources. Second, the narrow scope is what allowed for the in-depth study of the relationship between $\text{RI}_o$ and $\eta$. In addition to considering the effects of variable layer depths, and impeller-to-tank diameter ratio, the effects of different impeller designs, draft tubes, and inflows and outflows should all be considered.

**Notation**

The following symbols are used in this technical note:

- $a, b$: constants;
- $D$: impeller diameter;
- $D_T$: tank diameter;
- $g$: gravity;
- $g'$: reduced gravity;
- $H$: blade height;
- $L$: characteristic length scale;
- $N$: rotational rate (rps);
- $N_p$: power number;
- $P$: impeller power;
- $\text{RI}_o$: impeller Reynolds number;
- $\text{RI}_o$: overall Richardson number;
- $R$: impeller radius;
- $T_m$: mixing time;
- $t^*$: nondimensional time;
- $U$: characteristic velocity scale;
- $U_{tip}$: blade tip velocity;

- $z_i$: elevation of pycnocline;
- $z_i^*$: nondimensional elevation of pycnocline;
- $\Delta \rho$: density difference;
- $\eta$: mixing efficiency;
- $v$: kinematic viscosity;
- $\rho$: density; and
- $\omega$: rotational rate (rad/s).

**References**


