WATER RESOURCES RESEARCH, VOL. 45, W02501, doi:10.1029/2008WR007347, 2009

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Multiobjective sensitivity analysis to understand the information content in streamflow observations for distributed watershed modeling

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Received 9 August 2008; revised 3 November 2008; accepted 14 November 2008; published 4 February 2009.

[1] In a previous paper, van Werkhoven et al. (2008b) demonstrated that the information content of streamflow observations at a watershed outlet is a dynamic entity and is dependent on the spatiotemporal dynamics of the causal precipitation event. This result has important consequences for distributed hydrological model calibration strategies and for the design of observation networks. However, the conclusions drawn were based only on the analysis of the model parameter sensitivities to the hydrograph peak fit because of the use of the root-mean-square error objective function. An unanswered question is how will the previous result change if alternative objective functions are used? Here we extend the earlier analysis by adding low-flow and water balance objective functions. We study their impact on how much information can be extracted during calibration overall and for specific model components (parameters) using a synthetic rainfall-runoff event. Results suggest that both vertical (within a model cell) and spatial (across cells) sensitivities vary greatly with the objective function used. Timing-related objective functions show sensitivity largely focused on the area close to the outlet, while a volume-based objective function shows sensitivity distributed more evenly across the watershed. These results demonstrate the importance of using multiple evaluation metrics when assessing distributed model predictions. The resultant multiobjective sensitivity maps provide helpful tools for assessing the actual information provided by gauges in observation networks and motivate the need for a new generation of dynamic calibration strategies that would consider how the spatial parameter controls on the model response of interest vary in time.

Citation: Wagener, T., K. van Werkhoven, P. Reed, and Y. Tang (2009), Multiobjective sensitivity analysis to understand the information content in streamflow observations for distributed watershed modeling, *Water Resour. Res.*, *45*, W02501, doi:10.1029/2008WR007347.

1. Introduction

[2] The application of spatially distributed watershed models for hydrological research and operational applications is becoming increasingly common. The move from lumped to distributed watershed models requires us to rethink how we can best utilize point observations (most often streamflow) for the identification of spatially distributed parameters. While spatially distributed observations (e.g., through remote sensing) are becoming more and more available, point observations such as those of streamflow are often the only long-term observations of the watershed response available for calibration. Streamflow is thus likely to remain an important component of distributed model calibration.

[3] Most calibration strategies currently applied make assumptions about the spatial structure of the parameter field to reduce the often very high dimensionality of the parameter space of distributed models. Two main strategies that have been applied to achieve this dimension reduction are regularization and the use of multipliers on spatial parameter fields. Regularization is often applied in the form of Tikhonov regularization, in which constraints are added to reduce parameter variability through equations that relate parameters to each other or to preference values. These equations effectively reduce the search space by balancing the model's predictive performance with the complexity of the parameterization in space [e.g., *Tonkin and Doherty*, 2005; *Pokhrel et al.*, 2009]. In the case of multipliers, initial estimates of the spatially distributed model parameter values are usually derived from a priori available information first (e.g., soils data) [e.g., *Yatheendradas et al.*, 2008]. A multiplier specific to the spatial field of one parameter is then calibrated, assuming that the relative spatial difference between the parameters does not change. This reduces the dimensionality of the search space, though not necessarily its complexity, to that of an equivalent lumped model. In both cases, these approaches do not consider the impacts of distributed forcing when simplifying the models' distributed parameterizations.

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0043-1397/09/2008WR007347$09.00

[4] In two recent papers, *Tang et al.* [2007] and *van Werkhoven et al.* [2008a] took different paths to assess the value of streamflow observations at the watershed outlet for a typical grid-based model. Both used the same global variance–based sensitivity analysis to understand the sensitivity of the streamflow response at the watershed outlet to changes in the independent and spatially distributed model parameters. The result in both studies suggested a strong interaction between the spatial distribution of rainfall and parameter sensitivity.

[5] In this technical note, we will complement the recent results of *van Werkhoven et al.* [2008a] by testing how the use of additional objective functions modifies the spatial sensitivity pattern and how it changes the controlling parameters in each grid cell. The study setup is identical to the one used by *van Werkhoven et al.* [2008a] to allow for a direct comparison of results. This study only uses a uniformly distributed rainfall input since this model forcing is most likely to reveal differences between objective functions.

2. Methods, Model, and Data

[6] In this study we run a typical grid-based spatially distributed watershed model using synthetic data to evaluate model behavior in an error-free environment. The methods, model, and data used are described briefly below.

[7] Global sensitivity analysis approaches evaluate the sensitivity of the model output to changes in the model parameters across the full parameter space as defined by the modeler, i.e., not just around a reference point as local approaches do. Sobol''s sensitivity analysis method [*Sobol'*, 1993] is a variance-based approach that decomposes the model output variance into the relative contributions from individual parameters and from parameter interactions:

$$D(f) = \sum_i D_i + \sum_{i<j} D_{ij} + \sum_{i<j<k} D_{ijk} + D_{12\ldots,p}, \tag{1}$$

where p is the total number of parameters, f is the distribution of model output, $D(f)$ is the full model output variance, D_i is the output variance due to the ith component of the input parameter vector Θ, D_{ij} is the output variance due to the interaction of parameters θ_i and θ_j, and the final two terms represent third-order and greater interactions. In this study, each model parameter in each grid cell is treated as an independent parameter to fully assess the spatial distribution of sensitivity and thus to understand where information extracted from streamflow observations at the outlet might be useful for model calibration. The number of parameters per cell (17) times the total number of grid cells (78) leads to a high dimension of the parameter space (1326). The sensitivity of a parameter can be quantified through the ratio of its variance contribution to the full (i.e., due to all parameters) output variance. This ratio provides an index between 0 and 1. The "total" Sobol' sensitivity index used in this study reflects the combined effect of the parameter alone (i.e., individual sensitivity) plus its interactions with all other parameters in the analysis. For more details on Sobol''s method see *Sobol'* [1993, 2001], *Saltelli et al.* [1999], or *van Werkhoven et al.* [2008b].

[8] The model output variance with respect to streamflow is usually measured as the variance in an objective function. It is by now well established that a single objective function is insufficient to extract all available information from streamflow time series during model calibration [*Gupta et al.*, 1998]. Here we use high-flow, low-flow, and water balance–focused metrics to cover a range of streamflow characteristics. The high-flow metric is the commonly used root-mean-square error (RMSE), defined as

$$\mathrm{RMSE} = \sqrt{\frac{1}{m}\sum_{t=1}^{m}\left(\mathrm{Q}_{s,t} - \mathrm{Q}_{o,t}\right)^2}, \tag{2}$$

where m is the number of time steps, $Q_{s,t}$ is the simulated flow for time step t, and $Q_{o,t}$ is the observed flow in time step t. For the low-flow metric, the simulated and observed flow time series are first transformed by a Box-Cox transformation (equation (3)) with a λ value of 0.3, which has a similar effect as a log transformation. The RMSE of the transformed flows is then calculated to obtain a metric that emphasizes low flow, referred to here as the transformed root-mean-square error (TRMSE) (equation (4)):

$$Z = \frac{(1+Q)^{\lambda} - 1}{\lambda} \tag{3}$$

$$\mathrm{TRMSE} = \sqrt{\frac{1}{m}\sum_{t=1}^{m}\left(Z_{s,t} - Z_{o,t}\right)^2}, \tag{4}$$

where m is again the number of time steps, $Z_{s,t}$ is the transformed simulated flow for time step t, and $Z_{o,t}$ is the transformed observed flow in time step t. The final metric, the runoff coefficient error (ROCE), captures the overall accuracy of the water balance by first combining the flows into one characteristic hydrologic descriptor, the mean annual runoff coefficient. The absolute error in the runoff coefficient is then calculated, and thus, the ROCE is defined as

$$\mathrm{ROCE} = \left|\frac{\overline{Q}_s}{\overline{P}} - \frac{\overline{Q}_o}{\overline{P}}\right|, \tag{5}$$

where $\overline{Q}_s$ and $\overline{Q}_o$ are the simulated and observed mean annual runoff volume and $\overline{P}$ is the mean annual precipitation.

[9] The particular distributed watershed model applied is the National Weather Service's Hydrology Laboratory Distributed Hydrologic Modeling System (HL-DHMS) [*Koren et al.*, 2003, 2004]. HL-DHMS represents the watershed as 4 × 4 km grid cells which each consists of a water balance, a hillslope-routing, and a channel-routing component. Here, the Sacramento Soil Moisture Accounting Model (SAC-SMA) [*Burnash*, 1995] is used as the water-balance component (see parameters in Table 1), and the kinematic wave method is used for hillslope and channel routing. Both hillslope- and channel-routing parameters are not included in the sensitivity analysis since our previous results showed that the channel routing parameters in particular would otherwise dominate the sensitivity of the model. The parameter ranges used for the sensitivity analysis are shown in Table 1. The SAC-SMA is vertically separated into an upper and a lower zone, each consisting of tension and free-

Table 1. Description of SAC-SMA and Routing Parameter Descriptions, a Priori Grid Ranges, and Sensitivity Analysis Ranges

Parameter	Description	National Weather Service a Priori Grid Range	Sensitivity Analysis Range[a]
UZTWM	Upper zone tension water maximum storage (mm)	24–65	19.2–78
UZFWM	Upper zone free water maximum storage (mm)	11–54	8.8–64.8
UZK	Upper zone free water withdrawal rate (d^{-1})	0.19–0.76	0.152–0.912
PCTIM	Permanent impervious area (%/100)	0	0–0.05
ADIMP	Saturated (additional) impervious area (%/100)	0	0–0.2
RIVA	Area affected by riparian vegetation (%/100)	0	0–0.2
ZPERC	Maximum dry condition percolation rate	34–117	27.2–140.4
REXP	Percolation equation exponent (–)	2.11–2.89	1.69–3.47
PFREE	Percolation going to lower zone free water (%/100)	0.2–0.46	0.16–0.55
LZTWM	Lower zone tension water maximum storage (mm)	77–208	61.6–249.6
LZFPM	Lower zone free-water primary maximum storage (mm)	11–49	8.8–58.8
LZFSM	Lower zone free-water supplementary maximum storage (mm)	24–161	19.2–193.2
LZPK	Lower zone primary withdrawal rate	0.051–0.22	0.0408–0.264
LZSK	Lower zone supplementary withdrawal rate (d^{-1})	0.0021–0.0146	0.00168–0.0175

[a]Ranges adjusted on the basis of expanding the a priori ranges for the Blue River Basin by ±20%.

storage components acting as reservoirs to produce runoff and evapotranspiration. The zones are connected by a percolation function that calculates lower zone recharge depending on the actual storage content in both upper and lower zones. The model has additional parameters that are used to represent static and dynamic impervious areas.

[10] A synthetic error-free hourly time step rainfall-runoff data set was created using HL-DHMS and the physical characteristics of the Blue River Basin (1248 km^2) located in southern Oklahoma (Figures 1a and 1b). Rainfall is distributed uniformly across the watershed with a temporal distribution based on design storm hyetographs defined by the *U.S. Department of Agriculture Soil Conservation Service* [1986] for frontal events. We applied a maximum 24-hour precipitation accumulation (P_{tot}) of 157 mm on the basis of the 10-year return period for a 24-hour event for the basin location.

3. Results and Discussion

[11] Figure 1 summarizes the results of the sensitivity analysis for all three objective functions. Figures 1a and 1b show the rainfall-runoff data, and Figures 1c–1e show sensitivity maps (i.e., total aggregated sensitivities per grid cell) and the sensitive parameters (in gray) averaged across all grid cells in a schematic figure of the SAC-SMA model.

[12] Figure 1c shows the sensitivity analysis results with respect to the peak flow focused objective function, RMSE. In this case, the spatial sensitivity of the model is limited to the lower part of the watershed, while the parameters in the upper and middle parts hardly show any sensitivity within a cell. The parameters controlling the variance in the RMSE objective function are all located in the upper zone of the model, responsible for the quick runoff production in the cells. Sensitive parameters are the additional impervious area (ADIMP), which produces a quick runoff response, and the parameters defining the upper zone storage characteristics (UZTWM, UZFWM, and UZK), whose release represents an interflow-like response component. This result suggests that RMSE can be used to identify near-surface parameters but only relatively close to the gauging station. There is an interesting difference between the sensitivity to RMSE for event scales versus longer time scales. The latter was analyzed for the lumped SAC-SMA model by *van Werkhoven et al.* [2008b], who found that the controlling parameters were located in both the upper and the lower zone. This is not the case for the shorter time scale.

[13] The sensitivity map does not change considerably when the low-flow objective function, TRMSE, is used instead of RMSE (Figure 1d), though the parameter sensitivities farther away from the watershed outlet seem to increase a bit. However, the parameters that produce the sensitivity differ. Instead of only showing sensitivity in the upper zone parameters, additional parameters in the lower zone show up. The lower zone parameters dominate the recession characteristics and are thus more important for TRMSE, which is a better measure of how well the model describes the overall hydrograph. The idea of adding the Box-Cox transformation before calculating the RMSE stems from earlier work by *Sorooshian and Dracup* [1980], who attempted to reduce the problem of heteroscedascity, i.e., the changing variance of model errors with streamflow magnitude which leads to an overemphasis of fitting peak flows. In this analysis we find that both upper and lower zone storage parameters are sensitive to TRMSE, thus suggesting that this objective function provides a more balanced assessment of the overall fit than RMSE.

[14] Figure 1e shows the sensitivity map resulting from use of ROCE, which analyzes how well the model reproduces the water balance. One can clearly see that for this measure, which only assesses the volumes of water released by the model without consideration for timing of the release, all of the cells show sensitivity. This result suggests that it might be possible to reasonably identify storage capacities across the watersheds but that timing-related parameters are more difficult and are limited to areas close to the gauge. The parameters that show sensitivity in the upper zone for ROCE are different from the ones for RMSE and TRMSE. Only the tension water maximum (UZTWM, from which evaporative losses are extracted) and the recession constant (UZK, which defines the outflow rate from the free-water store) are sensitive since they will probably have the largest impact on the actual amount of runoff produced from the upper zone over the time period of the event analyzed. Sensitive lower zone parameters are the same as for the TRMSE measure.

[15] For all three measures, the percolation parameters are never sensitive. This result is in line with the general

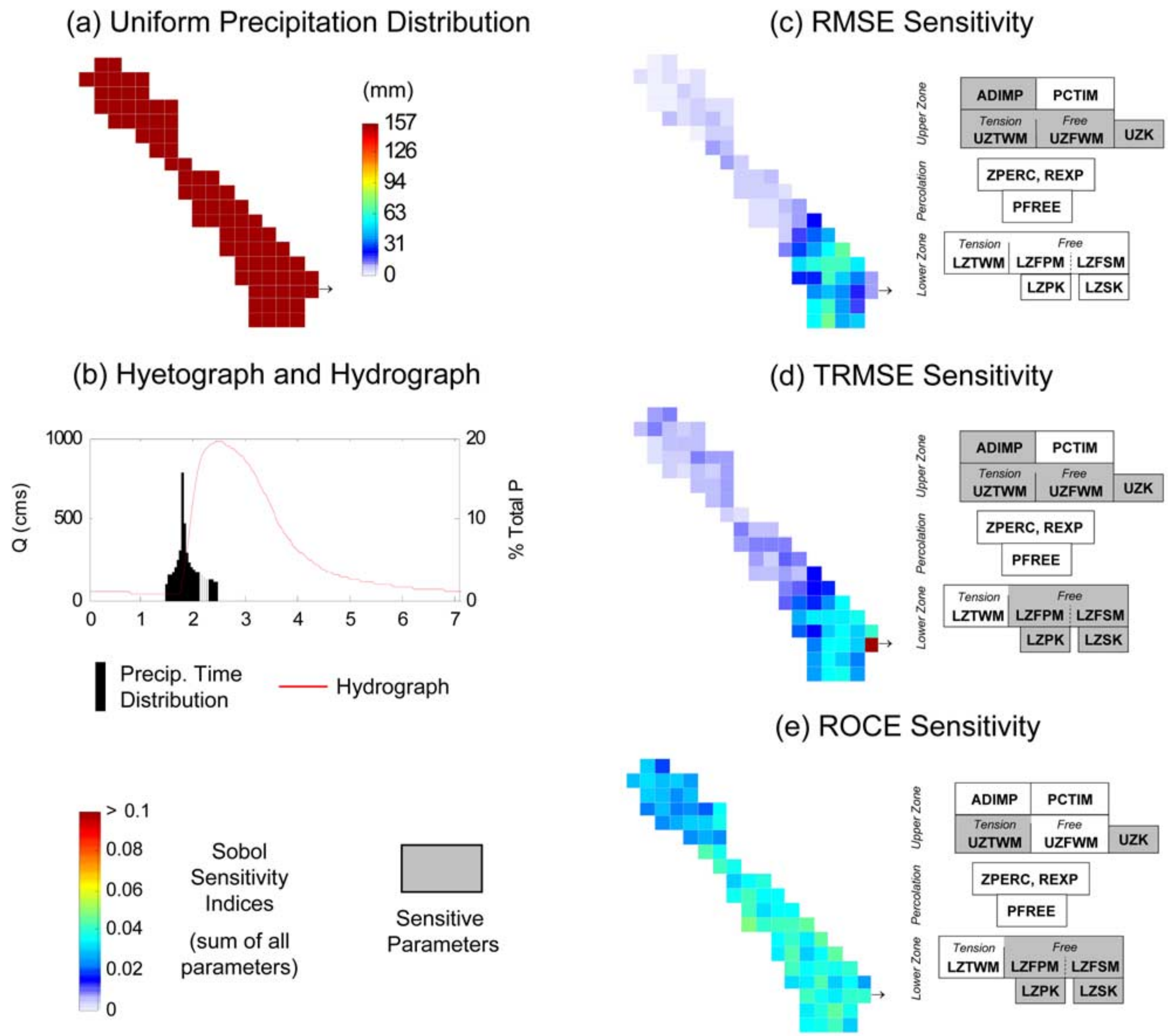

Figure 1. Multiobjective sensitivity analysis results. (a) Spatial uniformly distributed synthetic precipitation input across the grid-based watershed model domain. (b) Temporal distribution of rainfall and the resulting synthetic streamflow. (c–e) Sensitivities per cell as color map and dominant parameters highlighted in gray in the conceptual cell-based water balance model for three different objective functions.

experience during manual calibration of the Sacramento model in which the percolation parameters are hard to estimate.

4. Conclusions

[16] This study expands on earlier work of *van Werkhoven et al.* [2008a] by performing the first multiobjective sensitivity analysis of a spatially distributed watershed model using synthetic data for a single rainfall-runoff event. The results show that the choice of objective function has a significant impact on how the sensitivity is distributed spatially (across model grid cells) and vertically (within a single grid cell). The two objective functions for which timing and hydrograph shape are of concern, RMSE and TRMSE, are limited in their ability to extract information for grid cells farther away from the streamflow gauging station. The volume-based objective function, ROCE, on the other hand, leads to parametric sensitivities across the watershed.

[17] These results have direct relevance for guiding observational network design and for new calibration strategies tailored to distributed watershed models. Effective network design requires a clear understanding of the information provided by a new measurement point. The sensitivity analysis performed here demonstrates how the "cone of influence" of the information around a measurement point can be assessed using multiobjective spatial maps. This research also provides a first step toward a new generation of dynamic calibration strategies that will consider how spatial parameter controls on the model response of interest vary in time. For many time periods, large parts of the modeled watershed will not have significant para-

metric sensitivities; that is, no information is available to adjust these parameters to better values. Identifying these insensitive parameters in time has a strong potential to reduce the predictive biases that result in current calibration strategies which typically assume that all of the spatially distributed parameters across a modeled watershed are equally important during all time periods.

[18] **Acknowledgments.** Partial support for the first author was provided by SAHRA under NSF-STC grant EAR-664 9876800 and the National Weather Service Office of Hydrology under grants 665 NOAA/NA04NWS4620012 and NOAA/DG 133W-03-666 SE-0916. Support for the second author was provided by the Henry Luce Foundation in the form of a Clare Booth Luce Fellowship, a Penn State College of Engineering Fellowship, and a GE Faculty for the Future Fellowship. The third and fourth authors were partially supported by the National Science Foundation under grants EAR-0310122, EAR-0609791, and EAR-0609741. Any opinions, findings, and conclusions or recommendations expressed in this paper are those of the writers and do not necessarily reflect the views of any of the funding institutions.

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